

BCS theory of driven superconductivity

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Abstract. We study the impact of a time-dependent external driving of the lattice phonons in a minimal model of a BCS superconductor. Upon evaluating the driving-induced vertex corrections of the phonon-mediated electron-electron interaction, we show that parametric phonon driving can be used to elevate the critical temperature T_c , while a dipolar phonon drive has no effect. We provide simple analytic expressions for the enhancement factor of T_c . Furthermore, a mean-field analysis of a nonlinear phonon-phonon interaction also shows that phonon anharmonicities further amplify T_c . Our results hold universally for the large class of normal BCS superconductors.

1 Introduction

Quantum many-body systems which are driven far away from thermal equilibrium represent an increasingly fascinating realm of condensed matter physics, since recent progress in the experimental techniques has made it possible to manipulate condensed matter quantum states by strong external fields [1]. Light can strongly modify phases of correlated quantum many-body systems. For instance, strong time-dependent fields can induce transient superconducting phases in different material classes [2–8]. Moreover, electromagnetic irradiation can induce a collapse of long-range ordered charge-density wave phases [9–12], deconstruct insulating phases [13–15], or break up Cooper pair quasiparticles [16–19].

Conceptual insight into the possible physical mechanisms has been greatly advanced recently [20–26]. In the presence of strong lattice anharmonicities, the nonlinear coupling of a resonantly driven phonon to other Raman-active modes leads to a rectification of a directly excited infrared-active mode and to a net displacement of the crystal along the coordinate of all anharmonically coupled modes [20,21]. Selective vibrational excitation can also drive high- T_C cuprates into a transiently enhanced superconducting state. Moreover, on the basis of the non-equilibrium Keldysh formalism, partial melting of the superconducting phase by the pump field has been identified [22]. Furthermore, an advanced extension of the single-layer t - J - V model of cuprates to three dimensions has been used to show that an optical pump can suppress the charge order and enhance superconductivity [23]. In an effective approach on the basis of a driving-induced

reduction of the electronic hopping amplitude, the resulting increase of the density of states near the Fermi edge has been shown to enhance superconductivity [24]. Using the nonequilibrium dynamical mean-field theory for a strongly coupled electron-phonon system, a strong electron-mediated phonon-phonon interaction has been revealed [25]. These theoretical approaches are all very advanced and specialized to particular classes of systems and are rather successful in explaining experimental data for specific materials. Yet, it is still desirable to establish and analyze minimal models to reveal the fundamental mechanisms in terms of simple and elegant analytical results. Very recently, such a minimal model of a strongly driven electron-phonon Hamiltonian has been analyzed upon using Floquet formalism [26]. A Floquet BCS gap equation is derived which calls for a numerical solution and does not permit closed analytic results.

In this work, we aim to obtain a rather general and explicit analytical result to illustrate the driving-induced elevation of the critical temperature of a normal superconductor by extending the conventional BCS theory. We go beyond the existing approaches, which usually consider the modification of the distribution function of charge carriers (see e.g. [27]). We consider the standard Fröhlich-type electron-phonon Hamiltonian with linear phonons subject to a time-dependent external driving. We show that a simple dipolar coupling of the driving field to the phonon displacement coordinates only yields a scalar phase shift and does not modify the electron-phonon interaction vertex. In contrast to that, a parametric driving of the phonon frequencies strongly modifies the retarded Green's function, thereby changing the effective electron-electron attraction in a fundamental way. In order to quantify these effects, we introduce an elevation

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factor η of the critical temperature which can be directly calculated in our approach. In the limits of weak and strong driving fields, we obtain simple expressions for η , which reveal how the critical temperature can be enhanced even if the driving is nonresonant. Finally, we show that a parametric phonon drive combined with a phonon-phonon interaction can induce an additional elevation of the critical temperature. This is apparent already on the level of a mean-field treatment of the nonlinear phononics.

2 Minimal model of a driven BCS superconductor

The canonical modeling of the superconducting materials is based on the Fröhlich-type Hamiltonian ($\hbar = 1$)

$$H_0 = H_\psi[c_{\mathbf{k}}] + H_\Omega + \frac{\lambda}{\sqrt{V}} \sum_{\mathbf{k}, \mathbf{q}} (a_{\mathbf{q}}^\dagger + a_{-\mathbf{q}}) c_{\mathbf{k}-\mathbf{q}}^\dagger c_{\mathbf{k}}, \quad (1)$$

where $H_\psi[c_{\mathbf{k}}] = \sum_{\mathbf{k}} (\epsilon_{\mathbf{k}} - \mu) c_{\mathbf{k}}^\dagger c_{\mathbf{k}}$ is a Hamiltonian of the electronic conductance band,

$$H_\Omega = \sum_{\mathbf{q}} \Omega_{\mathbf{q}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}} \quad (2)$$

describes the phonon degrees of freedom with the dispersion $\Omega_{\mathbf{q}}$ and λ is the electron-phonon interaction strength. V is the volume of the sample. The deflection field $Q_{\mathbf{q}} = a_{\mathbf{q}}^\dagger + a_{-\mathbf{q}}$ of the phonons is the Fourier transform of the phonon coordinate. The BCS theory is build upon the fact that $Q_{\mathbf{q}}$ can be integrated over, such that an exact effective action

$$S = S_0 + \lambda^2 \sum_{\mathbf{q}, \mathbf{k}, \mathbf{k}'} \int dt dt' c_{\mathbf{k}+\mathbf{q}}^\dagger(t) c_{\mathbf{k}}(t) G(\mathbf{q}, t - t') \times c_{\mathbf{k}'-\mathbf{q}}^\dagger(t') c_{\mathbf{k}'}(t') \quad (3)$$

results. Here, $G(\mathbf{q}, t - t')$ is the Green's function (GF) of the deflection field $Q_{\mathbf{q}}$. Its retarded component is canonically defined as:

$$G^R(\mathbf{q}, \mathbf{q}', t - t') = -i\Theta(t - t') \times \langle Q_{\mathbf{q}}(t) Q_{\mathbf{q}'}(t') - Q_{\mathbf{q}'}(t') Q_{\mathbf{q}}(t) \rangle. \quad (4)$$

It generates an approximative interaction vertex amplitude $V(\mathbf{q}, \omega)$ of an effective electron-electron interaction mediated by the phonons and is at the heart of BCS theory of superconductivity. To lowest order in λ , one then obtains for the interaction vertex

$$V(\mathbf{q}, \omega) = \lambda^2 G_0^R(\mathbf{q}, -\mathbf{q}, \omega) = \frac{2\Omega_{\mathbf{q}}\lambda^2}{\omega^2 - \Omega_{\mathbf{q}}^2}, \quad (5)$$

where ω is the energy transfer during the scattering of the electron pair. Obviously, if $\omega^2 < \Omega_{\mathbf{q}}^2$, the effective interaction is attractive, thus leading to the Cooper instability and superconducting ground state [28]. In general, the larger the overall scale of $\Omega_{\mathbf{q}}$ is, the larger is the range of

energies ω and the higher is the number of electrons, for which the mutual interaction becomes attractive. This is accompanied by an increase of the critical temperature T_c , at which the superconducting gap vanishes.

The critical temperature in a BCS superconductor in a simplest model of an attractive constant potential of strength V_0 is given by ($k_B = 1$)

$$T_c \simeq \omega_D e^{-1/[V_0\rho(E_F)]}, \quad (6)$$

where $\rho(E_F)$ is the density of the electronic states at the Fermi edge and ω_D is the Debye frequency which fixes the characteristic energy scale for the phonon degrees of freedom. The expression in equation (6) can be considered as generic if one interprets V_0 as an effective parameter which measures the strength of the (in general energy and momentum dependent) attractive potential. There are basically three different options to increase T_c by changing one of the above parameters. We shall consider two of them: (i) the enhancement of the effective attraction V_0 ; and (ii) the increase of ω_D .

One way to modify the denominator of equation (5) is to drive the phonons by strong electromagnetic external THz fields. The driving can induce phonon excitations in sequential steps, in which the phonons are directly excited by applied EM field pulses. Alternatively, infrared-active phonon modes with a finite dipole moment can be excited, and due to nonlinear phonon coupling, normal phonon Raman modes of the crystal are excited [20,26]. We choose not to concentrate on these intricacies as they are strongly material-dependent and thus nonuniversal and consider the driving as acting directly on the relevant phonon mode. There are essentially two qualitatively different possibilities, the dipolar (or linear) driving where the drive couples to the phonon deflection field, and the parametric (or quadratic) driving where the drive modulates the phonon frequencies.

3 The effect of phonon driving

The dipolar phonon driving by an explicitly time-dependent driving field $\Delta_{\mathbf{q}}(t)$ does not influence the retarded GF $G^R(\mathbf{q}, \mathbf{q}', \omega)$. This immediately follows when we replace equation (2) by

$$H_\Omega = \sum_{\mathbf{q}} \Omega_{\mathbf{q}} a_{\mathbf{q}}^\dagger a_{\mathbf{q}} + \Delta_{\mathbf{q}}(t) (a_{\mathbf{q}}^\dagger + a_{-\mathbf{q}}). \quad (7)$$

The electron-phonon coupling strength λ is quite weak in most of the known superconducting materials. For this reason, the leading behavior of the $G^R(\mathbf{q}, \mathbf{q}', \omega)$ is dominated by the contribution of the phonon subsystem only. Solving the equations of motion for H_Ω , one readily finds $a_{\mathbf{q}}(t) = [a_{\mathbf{q}}(0) + f(t)]e^{-i\Omega_{\mathbf{q}}t}$, where

$$f(t) = -i \int^t dt' \Delta_{\mathbf{q}}(t') e^{i\Omega_{\mathbf{q}}t'}$$

is a simple time-dependent scalar phase shift. As the retarded GF is a commutator of fields, a mere shift of them

$$G^R(\mathbf{q}, \mathbf{q}', \omega) = i\delta_{-\mathbf{q}, \mathbf{q}'} \left\{ \frac{-i2\Omega_{\mathbf{q}}}{\omega^2 - \Omega_{\mathbf{q}}^2} - i \sum_{n=1}^{\infty} \frac{(\omega - \Omega_{\mathbf{q}})J_n \left[\frac{n\gamma}{\omega - \Omega_{\mathbf{q}}} \right]}{(\omega - \Omega_{\mathbf{q}})^2 - (n\Gamma_{\mathbf{q}}/2)^2} - \frac{(\omega + \Omega_{\mathbf{q}})J_n \left[-\frac{n\gamma}{\omega + \Omega_{\mathbf{q}}} \right]}{(\omega + \Omega_{\mathbf{q}})^2 - (n\Gamma_{\mathbf{q}}/2)^2} \right\}, \quad (11)$$

$$\bar{J}_n \left[\frac{n\gamma}{\omega - \Omega_{\mathbf{q}}} \right] = \frac{1}{2} \int_{-1}^1 dx J_n \left[\frac{xn\Delta_{\mathbf{q}}}{\omega - \Omega_{\mathbf{q}}} \right] \quad (13)$$

$$= \frac{1}{(n+1)!} \left[\frac{n\Delta_{\mathbf{q}}}{2(\omega - \Omega_{\mathbf{q}})} \right]^n {}_1F_2 \left[\frac{1+n}{2}; \frac{3+n}{2}, 1+n; - \left(\frac{n\Delta_{\mathbf{q}}}{2(\omega - \Omega_{\mathbf{q}})} \right)^2 \right] \quad (14)$$

does not affect the GF at all. Thus, we conclude that within our approximation the linear driving does not affect the conventional BCS superconductivity picture.

The parametric driving enters via the Hamiltonian

$$H_{\Omega} = \sum_{\mathbf{q}} [\Omega_{\mathbf{q}} + \Delta_{\mathbf{q}}(t)] a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}}. \quad (8)$$

It is, e.g., realized indirectly by resonantly driving infrared-active phonon modes with a finite dipole moment, which couple quadratically to normal Raman modes of the crystal [1,3,20,21] or by the quadrupole component of an electromagnetic field. The trivial case is the static driving, i.e., $\Delta_{\mathbf{q}}(t) = \Delta_{\mathbf{q}}$, which simply is an increase of phonon frequencies. As the Debye frequency rises as well, an increase of T_c is obvious. This effect is known and is experimentally detected in crystals subject to high pressure. In the dynamical case, the solution for the time evolution equation is obviously $a_{\mathbf{q}}(t) = a_{\mathbf{q}}(0)e^{-i\alpha(t)}$ with the phase $\alpha(t) = \Omega_{\mathbf{q}}t + \int_0^t dt' \Delta_{\mathbf{q}}(t')$. Then, the retarded GF of equation (4) follows as:

$$G^R(\mathbf{q}, \mathbf{q}', t, t') = -i\delta_{-\mathbf{q}, \mathbf{q}'} \Theta(t - t') \times \sum_{\pm} (\pm 1) e^{\pm i[\alpha(t) - \alpha(t')]} \quad (9)$$

Without restricting the generality, we henceforth assume periodic time-dependent driving in the form

$$\Delta_{\mathbf{q}}(t) = \Delta_{\mathbf{q}} \cos(\Gamma_{\mathbf{q}}t), \quad (10)$$

where $\Delta_{\mathbf{q}}$ is the strength and $\Gamma_{\mathbf{q}}$ is the frequency of the driving. After a Fourier expansion with respect to the time difference $t - t'$, we obtain a result in terms of the n th ordinary Bessel function [29]:

see equation (11) above

where

$$y = \Delta_{\mathbf{q}} \sin \left[\Gamma_{\mathbf{q}} \frac{(t + t')}{2} \right] \quad (12)$$

explicitly depends on the evolution time $\tau = (t + t')/2$. We recover the zero-order contribution of equation (5) as the first term of the r.h.s. of equation (11). Moreover, the multiphonon parametric resonances are apparent from the denominators when $2\tilde{\omega}_{\mathbf{q}} = n\Gamma_{\mathbf{q}}$.

To proceed, we exploit that the typical driving frequency in the experiments is in the THz regime, which is

slightly smaller than the typical Debye frequency of superconducting materials. Hence, we may average over the period of the external driving with respect to τ . For the time-averaged Bessel functions, we then obtain

see equations (13) and (14) above

for even n and zero otherwise. Here, ${}_1F_2$ denotes the hypergeometric function [29]. Its maximum is of the order of 1 for $n < 2$ for any argument and it decays exponentially for $n > 2$. Hence, we may focus on the lowest order term $n = 2$ only. The physical meaning is immediate. n denotes the number of phonons which participate in the renormalization of the GF by the vertex. The odd phonon numbers do not contribute for symmetry reasons. The larger n , the more efficient is the mutual cancellation during averaging. As a result, only the two-phonon process survives which is the parametric resonance.

In order to quantify the enhancement of the interaction vertex around the Fermi edge, we define the enhancement factor $\eta = G^R(\mathbf{q}, \mathbf{q}', 0)/G_0^R(\mathbf{q}, \mathbf{q}', 0)$ as the ratio of the two retarded GF in the low-energy limit. Moreover, we may exploit the asymptotic behavior of the hypergeometric function for $n = 2$ for $x \ll 1$ in the form $\frac{1}{6}x^2 {}_1F_2[3/2; 5/2, 3; -x^2] = x^2/6 + O(x^4)$ to assess the quantitative behavior of the GF in equation (4) in the vicinity of the Fermi edge $\omega \rightarrow 0$. Hence, for weak driving $\Delta_{\mathbf{q}} \ll \Omega_{\mathbf{q}}$, we obtain

$$\eta = \frac{G^R(\mathbf{q}, \mathbf{q}', 0)}{G_0^R(\mathbf{q}, \mathbf{q}', 0)} = 1 + \frac{\Delta_{\mathbf{q}}^2}{6(\Omega_{\mathbf{q}}^2 - \Gamma_{\mathbf{q}}^2)}. \quad (15)$$

$\eta > 1$ implies a relative enhancement of the attractive interaction around the Fermi edge and thus an increase in T_c , since η enters in the expression for the critical temperature as a factor renormalizing the electron-phonon coupling strength according to $V_0 \rightarrow \eta V_0$. This occurs for subresonant driving $\Omega_{\mathbf{q}} > \Gamma_{\mathbf{q}}$, which is the most realistic regime from the point of view of contemporary experiments, and can, at least in principle, become quite large. In the opposite case of superresonant driving $\Omega_{\mathbf{q}} < \Gamma_{\mathbf{q}}$, there is a decrease of T_c . This kind of transition should be experimentally observable.

In the limit of strong driving $\Delta_{\mathbf{q}} \gg \Omega_{\mathbf{q}}$, we obtain with $\frac{1}{6}x^2 {}_1F_2[3/2; 5/2, 3; -x^2] = 1/(2|x|) + O(1/x^2)$ for $x \gg 1$ the enhancement factor

$$\eta = 1 + \frac{\Omega_{\mathbf{q}}^3}{2\Delta_{\mathbf{q}}(\Omega_{\mathbf{q}}^2 - \Gamma_{\mathbf{q}}^2)}. \quad (16)$$

It shows the similar dependence of $\Omega_{\mathbf{q}}$ and $\Gamma_{\mathbf{q}}$. Although an estimate of the validity region of our approximation is more involved, we believe our results to hold for $\eta \sim 1-2$.

4 Nonlinear phononics

Next, we address the role of the phonon anharmonicity. On the microscopic level, it arises due to a nonlinear interaction between the phonons. Usually, one encounters two different kinds: three- and four-phonon interaction processes. They are described by the Hamiltonians

$$H_3 = \sum_{\mathbf{q}, \mathbf{k}} M_3(\mathbf{q}, \mathbf{k}) Q_{\mathbf{k}} Q_{\mathbf{q}} Q_{-\mathbf{k}-\mathbf{q}}, \quad (17)$$

$$H_4 = \sum_{\mathbf{q}, \mathbf{k}, \mathbf{p}} M_4(\mathbf{q}, \mathbf{k}, \mathbf{p}) Q_{\mathbf{k}} Q_{\mathbf{q}} Q_{-\mathbf{k}-\mathbf{p}} Q_{-\mathbf{q}+\mathbf{p}}, \quad (18)$$

where $M_{3,4}$ are the corresponding interaction amplitudes. As a rule, they are small and the appropriate way to assess their influence is the perturbation theory. It turns out that the three-phonon self-energy vanishes exactly for homogeneous systems and is strongly suppressed in lattices with high symmetry groups. Hence, we focus on the four-phonon process. We are interested in the effective properties of one single phonon mode. Therefore, the most important contribution is expected to be given by the non-diffractive scattering processes of the given phonon mode on itself, when $\mathbf{p} = 0$ and $\mathbf{k} = \mathbf{q}$. The underlying effective Hamiltonian [30] can be inferred from the above one and one finds

$$H_{\Omega} = \sum_{\mathbf{q}} \Omega_{\mathbf{q}} a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + \chi(\mathbf{q}) a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} a_{\mathbf{q}}. \quad (19)$$

The anharmonicity coefficient $\chi(\mathbf{q})$ can be obtained from $M_4(\mathbf{q}, \mathbf{q}, \mathbf{0})$ and is expected to be small. Since phonons at rest do not exist we can write $\chi(\mathbf{q}) \approx \chi_1 q$, where $q = |\mathbf{q}|$. Although in a superconducting material at low temperatures the phonon expectation value $\langle a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} \rangle = N_{\mathbf{q}}$ is strongly suppressed, this is not the case in presence of an external drive. Invoking a mean field approximation, the effective Hamiltonian is found to be

$$\begin{aligned} H_{\Omega} &\approx \sum_{\mathbf{q}} \Omega_{\mathbf{q}} a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + \chi(\mathbf{q}) a_{\mathbf{q}}^{\dagger} N_{\mathbf{q}} a_{\mathbf{q}} \\ &= \sum_{\mathbf{q}} [\Omega_{\mathbf{q}} + \chi(\mathbf{q}) N_{\mathbf{q}}] a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}}. \end{aligned} \quad (20)$$

In equilibrium and without external driving, $N_{\mathbf{q}}$ is determined from the self-consistency condition

$$N_{\mathbf{q}} = \left[e^{\beta(\Omega_{\mathbf{q}} + \chi_{\mathbf{q}} N_{\mathbf{q}})} - 1 \right]^{-1}, \quad (21)$$

and turns out to be smaller in comparison to the linear system with $\chi_{\mathbf{q}} = 0$. For this reason, the impact of the anharmonic phonon subsystem on the electronic properties is negligible and does not induce any appreciable change in T_c without driving [31]. This is completely different in

a strongly driven system, where the phonon population $N_{\mathbf{q}}$ is determined by the irradiation field. In this case, the phonon subsystem is stiffer and is characterized by an effectively enhanced Debye frequency ω_D . In order to illustrate this feature, we consider the simplest case $\Omega_{\mathbf{q}} = v_s q$, where v_s is the bare sound velocity of the crystal. Then, $\Omega_{\text{eff}}(\mathbf{q}) \equiv \Omega_{\mathbf{q}} + \chi(\mathbf{q}) N_{\mathbf{q}} \approx (v_s + \chi_1 N_{\mathbf{q}}) q$. Hence, the critical temperature is renormalized according to $T_c \rightarrow \xi T_c$ with $\xi = 1 + N_{\mathbf{q}} \chi_1 / v_s$ and is thus increased.

Hence, if a nonlinear superconducting material is exposed to strong external parametric driving, the critical temperature can be increased by two effects, so that equation (6) is modified to

$$T_c \simeq \xi \omega_D e^{-1/[\eta V_0 \rho(E_F)]}, \quad (22)$$

when $\eta, \xi > 1$. Overall, the theory is expected to hold quantitatively up to $\Delta_{\mathbf{q}} / \Gamma_{\mathbf{q}} \simeq 1$. It is important to realize that the enhancement factor enters in the exponent of T_c .

5 Discussion and conclusions

By considering a minimal model of a Fröhlich-type BCS Hamiltonian of a normal superconductor in presence of a time-dependent periodic electromagnetic driving of the phonons, we illustrate the basic physical mechanisms by which the critical temperature T_c can be elevated. We show that while a dipole (linear) driving cannot change T_c of the material, quadratic (parametric) driving can enhance the effective attractive phonon-mediated electron-electron interaction and thus increase the critical temperature. The effect in this minimal model is illustrated in terms of the enhancement factor of the interaction vertex caused by the external driving. In the limits of weak and strong external phonon driving, we find simple analytic results for the vertex enhancement. Furthermore, although an additional phonon anharmonicity does not change T_c in BCS superconductors held at equilibrium, nonlinear phononics can provide an additional contribution to the elevation of T_c , in that the effective Debye frequency is renormalized. Finally, we note that the external phonon drive also increases electron scattering, which in general suppresses Cooper pairing. Yet, it has been shown recently [26] that the dynamic enhancement of the formation of Cooper pairs addressed here dominates over the increase of the scattering rate. These results in terms of a minimal model shed new light on the essential ingredients needed for manipulating the characteristics of a BCS superconductor. A detailed analysis of the quasiparticle decay processes and their interplay with enhanced interaction vertex is an obvious avenue for further research [32]. Another source of T_c suppression might stem from thermal phonons, the effect of which can be taken into account along the lines of references [33,34]. In future work, one also could allow for more realistic interactions, as is, for instance, shown in reference [35]. These effects are, however, to a larger degree material-dependent and thus less universal.

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