Eur. Phys. J. Plus (2014) **129**: 198
DOI 10.1140/epjp/i2014-14198-7

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

MIXOPTIM: A tool for the evaluation and the optimization of the electricity mix in a territory

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Received: 11 March 2014 / Revised: 7 July 2014
Published online: 25 September 2014 –

Abstract. This article presents a method of calculation of the generation cost of a mixture of electricity sources, by means of a Monte Carlo simulation of the production output taking into account the fluctuations of the demand and the stochastic nature of the availability of the various power sources that compose the mix. This evaluation shows that for a given electricity mix, the cost has a non-linear dependence on the demand level. In the second part of the paper, we develop some considerations on the management of intermittence. We develop a method based on spectral decomposition of the imposed power fluctuations to calculate the minimal amount of the controlled power sources needed to follow these fluctuations. This can be converted into a viability criterion of the mix included in the MIXOPTIM software. In the third part of the paper, the MIXOPTIM cost evaluation method is applied to the multi-criteria optimization of the mix, according to three main criteria: the cost of the mix; its impact on climate in terms of CO_2 production; and the security of supply.

1 Introduction

The ongoing debate about energy in Europe has created the need for tools providing a performance estimate of the various energy options under discussion. The evaluation of the cost of an electricity mix is not as simple as it might appear at first sight. Several factors introduce complications in the system. Firstly, electricity cannot be stored in large amounts at the present time, and the demand, which fluctuates, must be met exactly and at all times by providing sources. Secondly, these sources are called upon in a specific order, within the limits of their availability. Thirdly, this availability itself is variable, especially for renewable sources like wind or solar energy. Moreover, the capacity of interconnexion between territories is growing, thus giving an increasing importance to imported and exported power fluxes (with wildly fluctuating prices). The conjunction of these four factors suggests a rather non-linear behavior of the system, and *a priori* hampers the determination of average values for cost evaluation.

The classical approach based on the use of averaged load factors (definition in sect. 2.6) is not fully satisfactory because it is not self-consistent: it uses load factors as input data while they should in principle be considered as output data, since they depend on the energy mix.

The economic performance of an electricity mix is not the only criterion by which this mix should be assessed. Growing worries about climate change incite one to consider the climate criterion also, especially the amount of CO_2 produced by the mix, taking into account the whole life cycle of the electricity produced by each source. Here again, the evaluation of the amount of CO_2 produced by the mix is a complicated matter, for the same reasons as presented above.

Last but not least, the choice of an electricity mix is also a political choice linked to a national energy strategy, where the energy independence of the territory must be considered. The degree of energy dependence of a given territory can be measured by the amount of electricity that must be imported from abroad, and by the probability that the demand exceeds the offer. Here also, the correct evaluation of these quantities pertaining to supply security calls for a simulation tool taking into account the fluctuating nature of the demand and of the supply.

The tool proposed here to simulate the behavior of the mix is a Monte Carlo simulation software, called MIXOPTIM.

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2 Simulating the behavior of an electricity mix using the Monte Carlo method

2.1 Data needed to characterize the electricity mix

We consider here a territory, with an electricity demand presenting fluctuations on the diurnal, weekly and seasonal timescales (fig. 1).

Even though it is quite predictable, this demand $D(t)$ is considered here as a random variable, characterized by a probability density law $p(D)$ (fig. 2) that can be derived from a sufficiently long historical sequence of the demand.

In order to satisfy the demand, the territory has an *in situ* production of electricity by means of several sources, hereafter labeled by the index i (wind, solar, hydraulic, nuclear, coal, gas, etc.) (fig. 3).

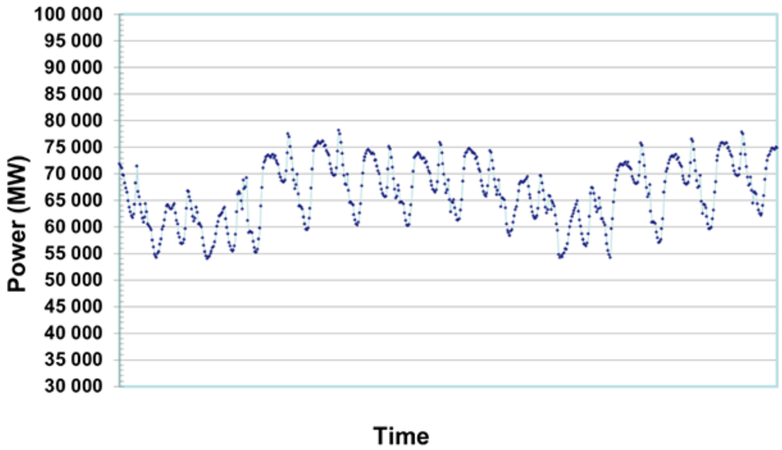

Fig. 1. Two weeks taken from the 2011 chronicle of the time dependence of the power demand on the French metropolitan territory (from ref. [1]).

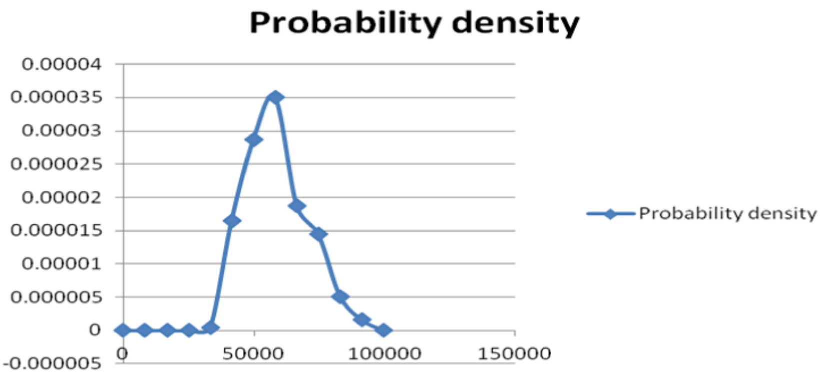

Fig. 2. The probability density $p(D)$ of the electricity demand in France (2011), from ref. [1].

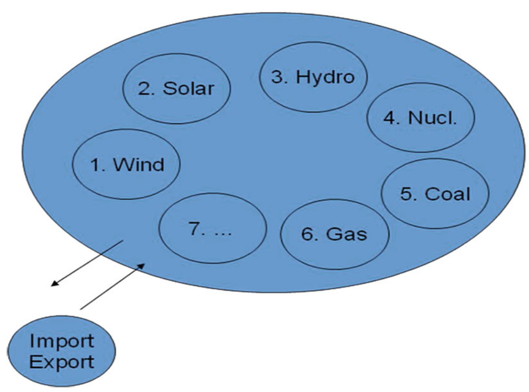

Fig. 3. The territory, and its power sources.

The use of these power sources can be characterized by the $(\alpha_1, \alpha_2, \ldots, \alpha_i, \ldots)$ and $(\beta_1, \beta_2, \ldots, \beta_i, \ldots)$ multiplets, where αi and βi are, respectively, the installed power and the power effectively produced by source i at time t. Both α and β are expressed in MW.

Moreover, the territory is not isolated, and can import or export power from outside (fig. 3).

Each source produces electricity at a given hourly cost C_i (expressed in €/h). At any given time t, the total cost can be split into two components:

$$C_i = F_i \cdot \alpha_i + M_i \cdot \beta_i.$$

Here, $F_i \cdot \alpha_i$ is the *fixed* cost of the source i (proportional to the *installed power* α_i), *i.e.* the cost that must be paid even if the source is not producing at time t. This cost component includes the amortization of the production installation, the cost of the personnel, the share cost of the distribution infrastructure, the insurance cost, etc... The fixed cost F_i is expressed in euros per unit installed power and per unit lifetime of the installation, *i.e.* in $€ \cdot \mathrm{MW}^{-1} \cdot \mathrm{h}^{-1}$.

$M_i \cdot \beta_i$ is the *variable* part of the cost that is proportional to the *power produced by the source*. This component includes for instance the cost of the fuel, operating costs, etc. The variable cost M_i is expressed in euros per produced energy, *i.e.* in €/MWh.

Table 1. The cost of some energy sources (€/MWh) from ref. [2].

	Wind	Solar	Hydraulic	Hydraulic pumping stations	Gen II LWR	Gen III LWR	Coal	Other renewables	Gas	Peak power sources (gas turbines)
Variable cost M	1	1	1	1	10	12	51	54	67	194
Fixed cost F	22	45	35	36	12	53	20	50	13	16

Table 1 gives the fixed and variable cost values used for the various sources in the present study. These cost values have been derived from a 2010 OECD report (ref. [2]). The fixed costs $F_i^{MIXOPTIM}$ used by MIXOPTIM must be adapted from this reference: in the OECD paper, as in most other papers on this subject, the hourly cost is indeed split into a fixed and a variable component, but with a definition different from the one taken in MIXOPTIM, since the hourly cost is taken for each source as proportional to the power produced by the source:

$$C_i = (F_i^{usual} + M_i^{usual}) \cdot \bar{\beta}_i = (F_i^{usual} + M_i^{usual}) \cdot K_{pi}^{assumed} \cdot \alpha_i.$$

This conventional definition of the cost requires the assumption of a value for the load factor $K_{pi}^{assumed}$. In order to retrieve an intrinsic fixed cost usable in MIXOPTIM without any assumption on the value of the load factor, we need to renormalize the fixed cost: $F_i^{MIXOPTIM} = F_i^{usual} \cdot K_{pi}^{assumed}$, while the variable cost can be taken unchanged:

$$M_i^{MIXOPTIM} = M_i^{usual}.$$

The amount of CO_2 produced by all power sources on the territory is evaluated with the same formalism, taking into account a fixed component f_i (proportional to the installed power) and a variable component m_i (proportional to the produced power) of the CO_2 production (see fig. 4). The fixed component includes the CO_2 produced during the life cycle of the power facilities. Both m_i and f_i are expressed in $\mathrm{KgCO_2/MWh}$ (table 2).

Note that this study does not take into account any carbon tax. A carbon tax can easily be added in MIXOPTIM by introducing a carbon contribution μ (expressed in $€/\mathrm{tCO_2}$) to the fixed and variable costs of each source: the transformation $F_i \to F_i + \mu \cdot f_i$; $M_i \to M_i + \mu \cdot m_i$ has been implemented in the software, with a μ-value chosen by the user.

Table 2. The CO_2 production of power sources in Kg/MWh (ref. [4]).

	Wind	Solar	Hydraulic	Nuclear	Oil	Coal	Gas
Variable contribution m	0	0	0	0	750	950	400
Fixed contribution f	18	70	7	2	150	50	100

2.2 Availability of the sources

The power sources on the territory are never fully available at time t: some wind turbines may be windless, some power plants may be out of operation or under maintenance. One defines $X_i(t)$, proportion of the source i that is *available* at time t $(0 < X_i(t) < 1)$.

Even though this source availability is generally quite predictable, the quantities $X_i(t)$ are considered in MIXOPTIM as random variables, characterized by a probability density $\pi_i'(X_i)$, where $\pi_i'(X_i) \cdot \mathrm{d}X_i$ is the probability to have at any time t, the available proportion of source i within the interval $[X_i, X_i + \mathrm{d}X_i]$ (see fig. 4).

It will be assumed hereafter that the random variables X_i and demand D are *independent*; for the sake of simplicity, MIXOPTIM neglects the correlations between these variables. There are, however, a few important correlations that must be taken into account. For example, the peak demand occurs generally during winter evenings, at a time when the photovoltaic source is unavailable. These correlations are treated in MIXOPTIM by considering separately four combined time periods: night-day/winter-summer. The probability laws for the demand and for the availability of the sources take different values in the four cases. The MIXOPTIM overall results are averaged over these four time periods.

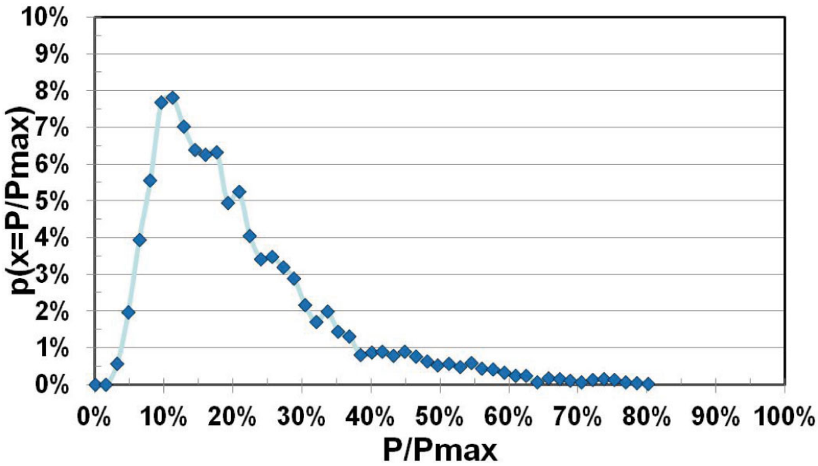

Fig. 4. Probability of availability of the French wind fleet (2011) (ref. [3]).

2.3 Order of solicitation of the sources

The sources are ranked by solicitation priority order.

Three rationales can be considered to determine the order of solicitation of the sources:

– a frequency rationale: baseload sources are mobilized first, followed by semi-base sources, then peak sources, in the order of rapidity of reaction of the sources.

– an economic rationale: the sources are solicited following the order of increasing variable cost, within the limits of their availability, and until satisfaction of the demand.

– a political rationale: certain sources are solicited in priority, by an *a priori* decision. For example, in Western countries, wind and solar power are presently considered as "mandatory" (the usual —very inadequate— term is "non-dispatchable"), and called systematically at their maximum available capacity.

MIXOPTIM considers a mixture of these three rationales, by imposing a fixed order of solicitation of the sources, chosen by the user. Generally, one solicits first the "mandatory" sources, then the remaining sources according to the economic rationale, and, when necessary, fast sources, even if they are expensive.

With the notable exception of combustion turbines, "controllable" (the usual term is "dispatchable") sources (nuclear, gas, coal) have a similar rapidity of reaction. Therefore, they may not be solicited very differently from each other to manage the follow-up of the intermittence. For these three sources, the economic rationale prevails.

In the case of wind and solar power, the economic and political rationales coincide, since these sources are generally considered as mandatory, and have a quasi-zero variable cost.

The case of gas, coal and fuel oil is special: one part of the installed power corresponds to industrial facilities working in cogeneration with a mandatory production, and another part of the installed power corresponds to controllable power sources, dedicated exclusively to electricity production. These two parts can be distinguished in MIXOPTIM by creating two sources placed differently in the source ranking order.

The case of hydraulic power is even more complicated. This source can be solicited for the load follow-up, because its cutoff frequency is very high. According to the frequency rationale, this source should be solicited last. However, according to the economic rationale, it should be solicited among the first, because its variable cost is almost zero. To solve this contradiction, MIXOPTIM can take into account three kinds of hydraulic sources: a mandatory hydraulic source (*e.g.*, run of the river), a semi-base hydraulic source (*e.g.*, lake hydraulic) placed midway in the ranking order, and a peak hydraulic source (pumping stations) placed last in the ranking order.

After having determined in MIXOPTIM the ranking order of the sources, one then determines the instantaneous mix $(\beta_1, \beta_2, \ldots, \beta_i, \ldots)$ which solicits the source i in the order previously defined, within the limits of their capacity available at time t, until satisfaction of the demand (table 3).

This instantaneous mix must satisfy the demand at any time t:

$$D(t) = \sum_i \beta_i(t); \qquad \beta_i(t) = x_i(t) \cdot \alpha_i.$$

Here, $x_i(t)$ is the proportion of source i that is *actually used* at time t.

Of course, this proportion is smaller than the available proportion, so one has $x_i(t) < X_i(t)$.

Table 3. Example of source solicitation by priority order.

Available proportion X_i	$X_1 = 0.25$	$X_2 = 0.70$	$X_3 = 0.60$	$X_4 = 0.90$
Proportion effectively used x_i	$x_1 = 0.25$	$x_2 = 0.70$	$x_3 = 0.30$	$x_4 = 0$
	Sources 1 and 2 are used within the limits of their availability		Source 3 provides the complement to satisfy the demand	Source 4 is not solicited

2.4 Power import, export and storage

The MIXOPTIM Monte Carlo software calculates at time t the random value of the power demand $D(t)$ according to the probability law $p(D)$, and the random values of the available proportions of each source i according to the probability laws $\pi'_i(X_i)$.

The import and export of energy to and from the territory are treated as all other sources: they contribute, respectively, negatively and positively to the power and financial balance of the territory. The imported and exported powers are limited by the interconnexion capacity α_{ie} of the territory. The price of imported/exported power is considered as a random variable characterized by a probability law taken for example from the statistics of POWERNEXT (ref. [5]). This price determines the ranking of the import/export in the solicitation priority order of the sources (fig. 5).

The territory can export some power, at an assumed export price, considered here as a random variable. All sources which have a marginal production cost lower than this export price are assumed to produce at full capacity, within the limits of their availability. The surplus power is exported and the benefits are computed by MIXOPTIM, and taken into account in the evaluation of electricity cost.

The territory may be able to store a limited amount of energy, and to give it back at very short notice to help satisfy sharp demand peaks. Presently, the main technology used for that purpose are hydraulic pumping stations; MIXOPTIM considers this storage capacity as a special hydraulic resource, characterized by a high availability law. The economic impact of these electricity storage facilities will be assessed in a forthcoming paper.

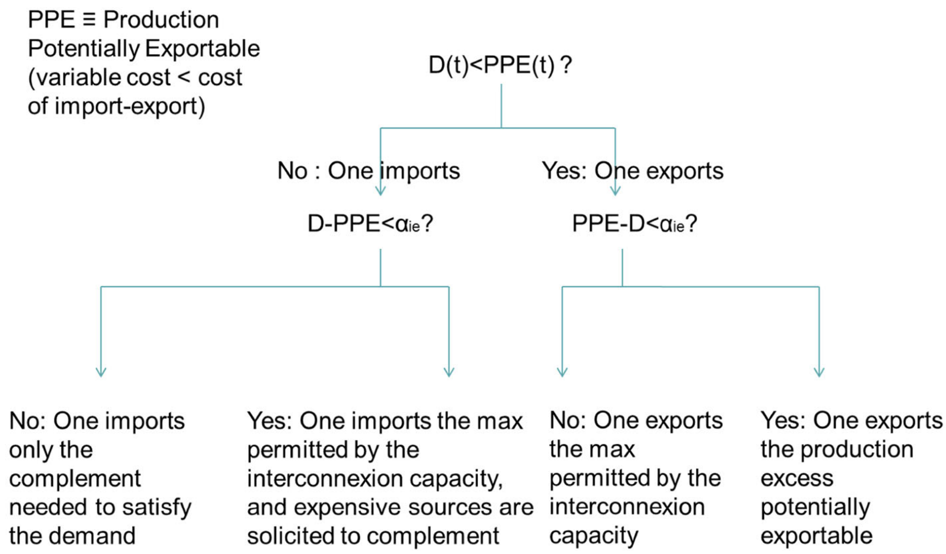

Fig. 5. The position of import-export in the ranking of the power sources. MIXOPTIM considers four possible cases for the respective values of the production and of the demand.

2.5 Power cuts

The sources are solicited according to the priority order, within the limits of availability defined above, and until the demand is satisfied. If the demand exceeds the total power production available on the territory at time t, including the maximum allowable imports, then the transmission system operator must proceed to load reduction or power shutdown. The MIXOPTIM software calculates the frequency of this event.

2.6 Calculation of the time-averaged performance of the mix

The performance of the mix at time t for the set of random variables D and X_i can be evaluated according to the above rules.

Firstly, MIXOPTIM calculates the load (or capacity) factors of all sources (contrary to most other evaluation tools, where these load factors are assumed).

For each source, the load factor is defined as the ratio of the energy output of a plant to the energy that could be produced if the plant operated at its nameplate capacity

$$Kp_i = \frac{\bar{\beta}_i}{\alpha_i} = \bar{x}_i.$$

Secondly, MIXOPTIM calculates indicators of performance of the mix. For example, the production cost at time t is

$$M(t) = \sum_i M_i \cdot \alpha_i \cdot x_i(t).$$

A large number of Monte Carlo (MC) draws is performed in order to provide a correct average value $\bar{M}$ of the production cost of the installed mix. Statistical convergence towards the real production cost averaged over long timescales is checked using standard MC numerical techniques.

The fixed cost can also be evaluated:

$$F = \sum_i F_i \cdot \alpha_i,$$

and the total cost, averaged over long timescales is

$$\bar{C} = \bar{M} + F.$$

An economic performance indicator of the mix can then be calculated:

$$P_{economy} = \frac{\bar{C}}{\bar{D}},$$

expressed in €/MWh.

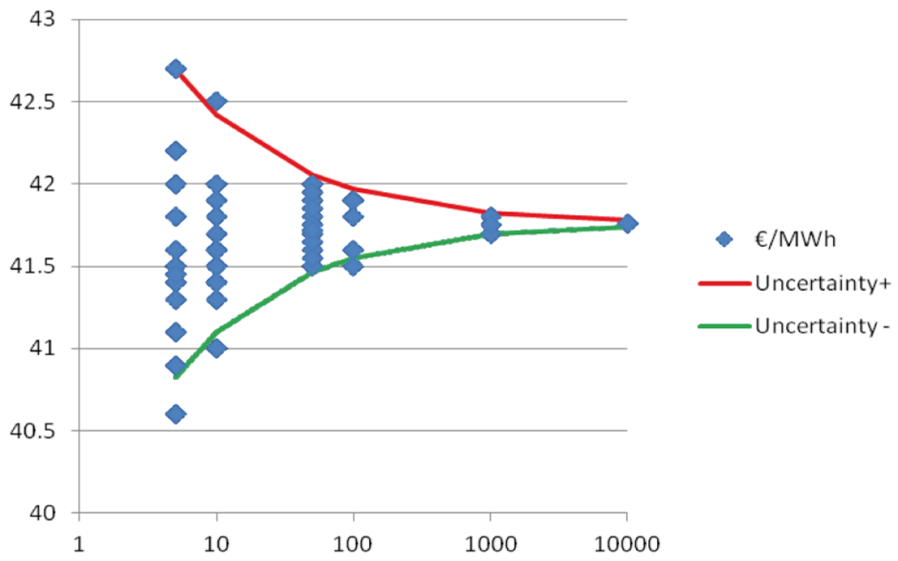

Fig. 6. Precision of the MC results as a function of the number of iterations. The region enclosed between red and green lines corresponds to a 90% confidence level. On a standard 2013 personal computer, evaluation of a mix with a precision of 0.4% on the cost requires 1000 MC iterations, and takes a few seconds. With 100 MC iterations, the precision is still of the order of 1.2%. More MC iterations are generally needed to obtain a good statistical significance on the frequency of power cuts.

The evaluation of the averaged CO_2 production of the mix can be made with the same method, and a climate performance indicator of the mix is calculated:

$$P_{\mathrm{CO}_2} = \frac{\overline{\mathrm{CO}_2}}{\bar{D}},$$

expressed in tons of CO_2/MWh.

The evaluation of the average imported power IMP and the frequency of power cuts FCUT is also provided by the MIXOPTIM software, and an indicator of supply security is calculated:

$$P_{supply} = \frac{\overline{IMP}}{\bar{D}} + FCUT.$$

Convergence tests show that a precision of 0.4% on the cost of the mix can be reached with 1000 Monte Carlo iterations in each demand bin (fig. 6).

2.7 First elements of validation of MIXOPTIM: results of the performance evaluation of the mix

MIXOPTIM has been validated by confronting its predictions to the actual results of an existing mix, *i.e.* the French mix 2011 (see table 5).

The performance of the 2011 French mix calculated by MIXOPTIM is reasonably close to the observed figures, except perhaps the amount of exported and imported power. In fact, MIXOPTIM simulates the power imports and exports in a rather crude way, as a simple stock exchange mechanism determined by a random value of the price of the exchanged MWh, and limited by the interconnection capacity. Even though this simulation probably describes rather well the short-term "spot" market, import-export exchanges are also ruled by a mid- and long-term market, not described in MIXOPTIM. This is probably the reason for the discrepancy between the observed and the calculated values.

The load-factors calculated by MIXOPTIM are important output results, that can be confronted with the actual values (see table 4).

The load factor of coal and gas sources is not perfectly reproduced, probably because it depends strongly on the proportion of these sources that must be considered as "mandatory", and because this proportion is not well documented.

The load factor of each source depends on the considered mix. For example, if one introduces a lot of intermittent mandatory sources, the load factor of the other sources will decrease. This mutual influence effect has been described in detail by Wagner [7]. A quantitative illustration is shown in fig. 7.

Table 4. The load factor of the sources in MIXOPTIM (case of the French mix 2011) (from ref. [6]).

	Wind	Sol	Coal	Gas	Nucl
Observed (2011)	0.21	0.11	0.19	0.40	0.79
Calculated	0.21	0.12	0.17	0.30	0.70

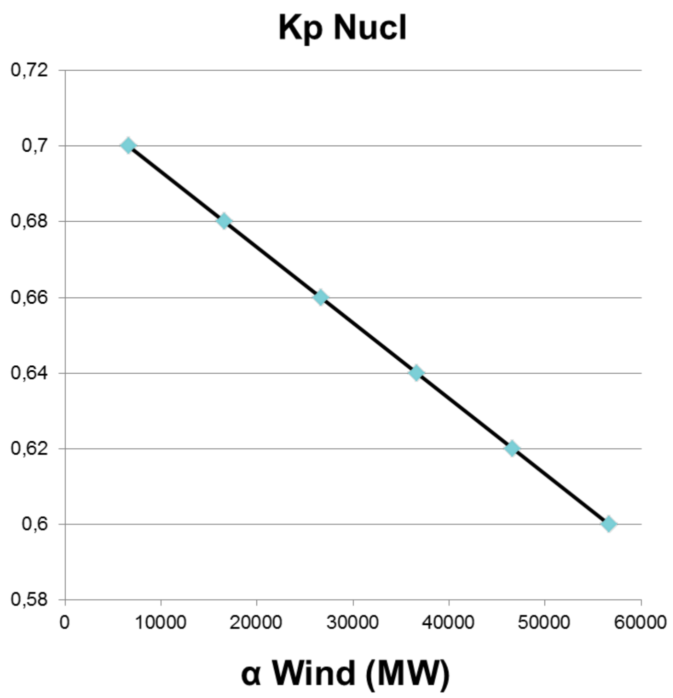

Fig. 7. Evolution of the load factor of the nuclear plants of the French mix 2011 as a function of an increasing value of the installed wind power. There is a mutual influence between sources.

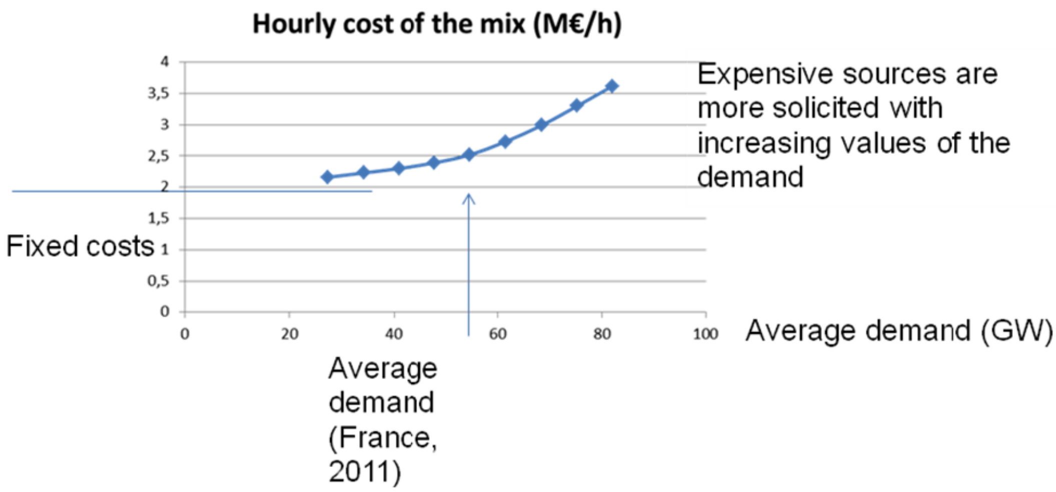

Fig. 8. Hourly cost of the mix in M€/h as a function of the average demand (French mix, 2011).

Table 5. Comparison between the observed and calculated performances of the French mix 2011 (from ref. [6]).

	Average demand (MW)	Cost (€/MWh)	CO_2 (Kg/MWh)	P imported (MW)	P exported (MW)
Observed (2011)	57 740	44.7	56.4	1 630	7 970
Calculated	54720	46.1	60.6	660	11950

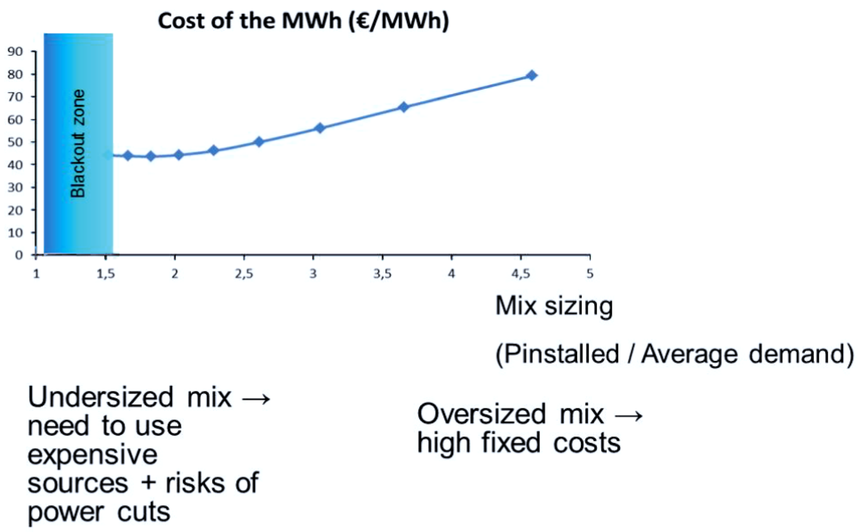

Fig. 9. Cost of the MWh of the mix as a function of the mix sizing. The proportion of sources in the mix is the one of the French mix 2011.

For a given mix, a growing demand requires the solicitation of more and more expensive electricity sources. Consequently, the cost associated with the satisfaction of the demand is far from proportional to this demand (see fig. 8). On the left of the diagram (low demand, oversized mix), the cost of the mix becomes roughly a constant determined only by the fixed costs. On the right of the diagram (high demand, undersized mix), the cost increases rapidly, because of the increasing contribution of the variable costs. On the extreme right, the mix is no longer able to satisfy the demand, and power cuts occur. This (expected) result confirms the non-linear behavior of the system.

For a given mix, the cost of MWh of the mix presents a minimum as a function of the installed power: if the installed power is too large, the fixed costs dominate the total and become overwhelming; if the installed power is too small, one frequently needs to solicit expensive energy sources (fig. 9). The economic optimum seems to require an installed capacity far beyond the average power demand.

This curve shows the existence of an optimally dimensioned mix, between an oversized mix, burdened by large fixed costs, and an undersized mix, where expensive sources are mobilized often, with high risks of power cuts.

The performance of the mix depends in principle on the probability laws used as input in the model, both for the power demand and for the source availability. These laws depend on the territory size. For a very large territory, one can expect rather regular and smooth probability laws. The performance of the mix therefore depends on the territory size, and on its characteristics. We thus predict different optimal mixes for different sizes of territory.

Although it is technically feasible to include the cost of the electricity transport and distribution in MIXOPTIM, as a special additional contribution to the fixed cost of each source, this has not been done in the present study.

The cost dataset used in MIXOPTIM could also include the contribution of externalities, such as those stemming from the health impacts of producing, distributing, and consuming energy, but here again these externalities have not been considered.

In the same vein, it is possible to study with MIXOPTIM the influence of a carbon tax on the cost of the MWh, but in the present study, the cost of carbon has been put to zero.

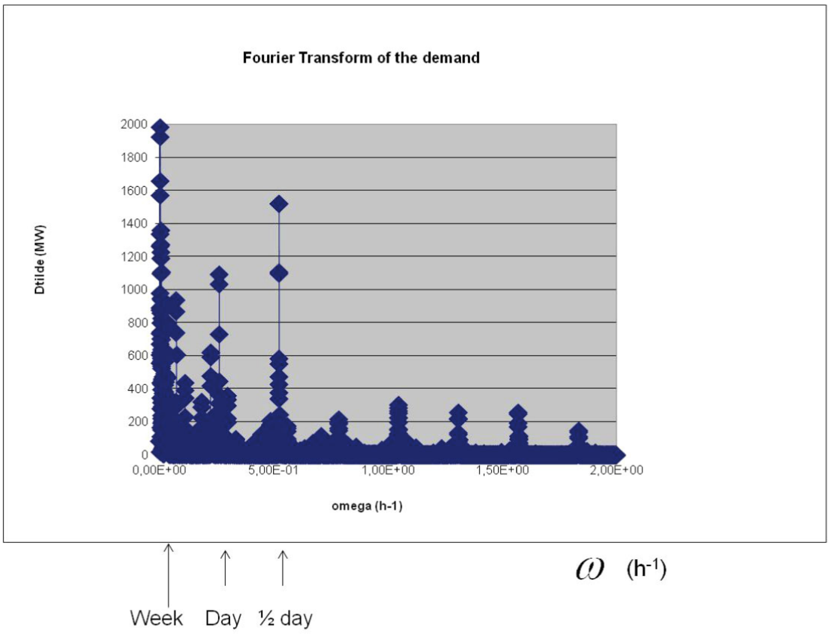

Fig. 10. The Fourier spectrum of the demand (France, 2011).

3 Intermittence management and flexibility of the sources

In this second part of the paper, we develop considerations on the management of the intermittence. The general trend of electricity mixes, at least in western countries, is an increase of the proportion of renewable, intermittent sources in the mix. This raises the question of the amount of controlled backup power needed to compensate for the fluctuations of these sources.

The mix evaluated by MIXOPTIM may be satisfactory from the point of view of its indicators of performance, but nothing indicates that it will be able to follow the fluctuations of the load. MIXOPTIM does not say anything about this, because MIXOPTIM does not treat the time dependence of the source management explicitly. However, MIXOPTIM contains a subroutine enabling one to assess the load-follow-up capacity of a given mix. The underlying physics is as follows.

Power fluctuations can be divided into two categories: on the one hand, the imposed fluctuations, like the fluctuations of the demand and the fluctuations of the power produced by intermittent, mandatory sources; on the other hand, the fluctuations of the controlled sources, which (in the absence of electricity storage facilities) must compensate exactly for the former ones.

The relationship

$$D - \sum_i P_{imposed} = \sum P_{controlled}$$

expresses this dichotomy between "imposed" fluctuations,

$$S = D - \sum_i P_{imposed},$$

and "controlled" fluctuations,

$$V = \sum_i P_{controlled}.$$

A useful tool for the evaluation of the intensity of the fluctuations is the Fourier transform of this equation:

$$\tilde{F}(\omega) = FT(F) = 1/2T \cdot \int_{-T}^{T} F(t) e^{i\omega t} \mathrm{d}t = FT\left(D - \sum_i P_{imposed}\right) = FT\left(\sum P_{controlled}\right),$$

where ω is the angular frequency, expressed here in h^{-1}.

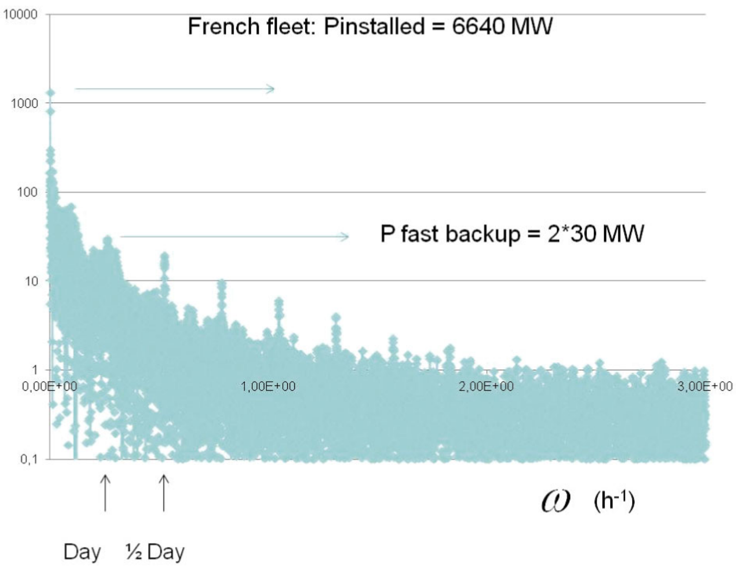

Fig. 11. The spectrum of the fluctuations of the wind production (French fleet 2011). As shown by the peaks at omega = 0.25 and $0.5\,\mathrm{h}^{-1}$, the wind production has a diurnal and semi-diurnal periodicity, probably linked to the existence of thermal breezes. The other peaks at higher frequency correspond to the harmonics of these two.

The Fourier transform of D and $P_{imposed}$ has been calculated for the French mix (year 2011). On the spectrum of the demand (fig. 10), one can observe the peaks at $\omega = 0.037$, 0.26, $0.52\,\mathrm{h}^{-1}$, corresponding to the weekly, daily and diurnal periodicities, respectively.

The spectrum also shows that the minimum power needed to follow the semi-diurnal fluctuations is about $2 \times 1600\,\mathrm{MW}$. The power needed to follow the higher frequency fluctuations is about $2 \times 400\,\mathrm{MW}$.

These spectra give information on the amount of power needed to compensate for the intermittence of the production of the mandatory sources, and on the frequency at which this power must be produced, via controlled sources. For example, fig. 11 shows that the minimum power needed to compensate for the intermittence of the (2011) French wind power fleet at very low frequency (for example during an anticyclonic episode, $\omega = 0.015\,\mathrm{h}^{-1}$) is of the order of $2 * 1500\,\mathrm{MW}$; it is the minimum power needed to back up the wind source; the power needed to back up the diurnal and semi-diurnal fluctuations is much smaller, of the order of $2 * 30\,\mathrm{MW}$.

The satisfaction of a rapidly fluctuating demand cannot be obtained with slowly responding sources. Each power source can be characterized by the time it takes to mobilize it or to modify its power production. For example, nuclear power plants can vary their production by about $2\%\,\mathrm{min}^{-1}$ (some management modes can be even faster, we give here only an order of magnitude). This value is about the same for coal power plants or combined cycle gas turbines, but is definitely higher for combustion turbines. These figures can be translated into a cutoff angular frequency ω_c (see table 6):

$$\omega_c = \frac{1}{P} \cdot \frac{\mathrm{d}P}{\mathrm{d}t}.$$

Above this characteristic frequency, the source cannot follow the time fluctuations of the demand.

Table 6. The cutoff angular frequency ω_c of the sources (h^{-1}) (order of magnitude).

Hydro	Wind	Solar PV	Combustion turbines	Nuclear	Gas	Coal
100	10	10	10	1	0.5	0.25

One can then define a power-frequency domain for each source as a zone inside which the source can be solicited. The frequency spectrum of the imposed fluctuations, calculated as the Fourier transform of $S(t)$, must then be enveloped by the sum of the frequency response functions of its controlled sources (fig. 12). Fulfilling this condition is a necessary condition (although not a sufficient one) to be able to follow the demand fluctuations. For every mix, MIXOPTIM checks whether this condition is satisfied or not.

We propose below an indicator of the fragility of the grid, based on the comparison between the magnitude of the imposed fluctuations and the capacity of the mix to compensate for these fluctuations.

MIXOPTIM checks that the spectrum of the imposed fluctuations is enveloped by the sum of the frequency response functions of all controllable sources in the mix (fig. 12). If this condition is not met, the mix is rejected as unable to react fast enough and powerfully enough to the imposed fluctuations. The larger the envelope, the stronger the mix will be.

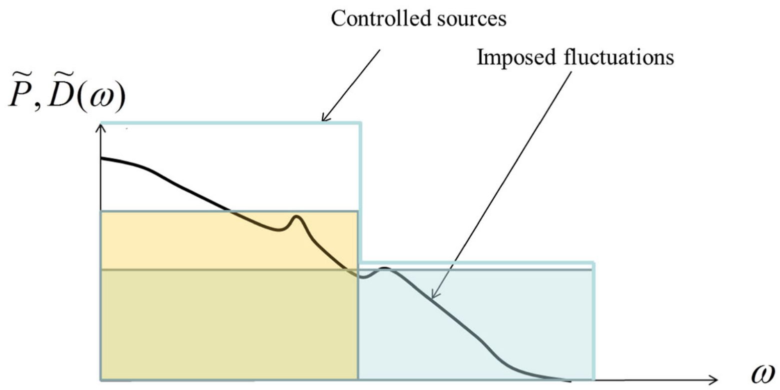

Fig. 12. The criterion utilized in MIXOPTIM for assessing the load following capability of a given mix.

The power demand fluctuates, as well as the mandatory power sources. In a first order approximation, these fluctuations can be considered as independent, and their relative phase is thus random. This means that one can find cases where a peak of the demand coincides with a low production of the mandatory sources (for example, an anticyclonic spell with no wind may coincide with a cold wave associated with a high value of the demand). This kind of consideration brings us to consider a more stringent condition than the one discussed above: Not only the spectrum of $S(t)$ must be enveloped by the response functions of the controllable sources,

$$\sum_{controlled} \alpha_i \cdot Kd_i \cdot c(\omega_{ci}) > 2 \cdot \tilde{S}(\omega)$$

but

$$\sum_{controlled} \alpha_i \cdot Kd_i \cdot c(\omega_{ci}) > 2\tilde{D}(\omega) + 2 \sum_{mandatory} \tilde{P}_i(\omega) \qquad \text{(criterion 1)}$$

in order to take into account the possible anticoincidence of the fluctuations of the demand and of mandatory sources.

In the case of France, this criterion will ask for the mobilization of at least $2 * 1600\,\text{MW}$ to follow the diurnal fluctuations, and $2 \times 2000\,\text{MW}$ of slow backup to compensate for the fluctuations of mandatory sources. These values are small because the French mix is still poor in renewable variable mandatory sources, but might increase rapidly if a large amount of wind and solar power is fed into the grid.

The import-export has not been included in the criterion, because this "source" can be considered either as a controllable source (available on request, in which case it should appear on the left side of the inequality) or as a mandatory source, whose fluctuations are imposed by the neighbour territories (in which case it should appear on the right side of the inequality). In our present state of uncertainty, we did not include it at all.

In order to have a mix able to satisfy the demand without too much relying on imports, one also needs to add the following criterion:

$$\sum_{All\ sources} \alpha_i \cdot Kd_i + \alpha_{imp\,\text{exp}} > D_{\max} \qquad \text{(criterion 2).}$$

This is another necessary (but not sufficient) condition for the independence of a territory.

The present French mix fulfils these two criteria hands down (fig. 13). However, if one decreases the part of the controlled sources in the mix, and if one replaces them by intermittent, mandatory sources, one gets rapidly close to the limit of criterion 1.

These two inequality criteria have been introduced into MIXOPTIM, under the form of a simple warning when MIXOPTIM is used for the evaluation of a simple mix, and under the form of an optimization constraint when MIXOPTIM is used for optimization.

It should be noted that no technical constraint obliges one to consider wind or solar sources as mandatory. If the wind or photovoltaic production at time t is too large for the needs of the territory and difficult to export, it is technically always possible to discard this power or avoid its production. It is only political constraints, transcribed into commercial obligations, which oblige the grid manager to put this excess power on the grid.

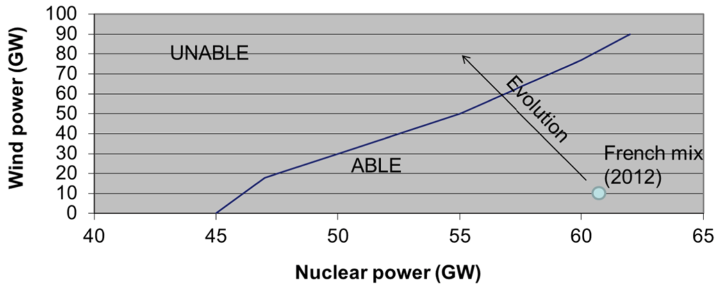

Fig. 13. The demarcation between able and unable mixes, according to the load follow-up criteria, as calculated by MIXOPTIM. Starting from the French case (2011), variations of the installed nuclear and wind power have been allowed. The real 2011 mix fulfils the follow-up capacity criteria easily, but the foreseen introduction of increasing amounts of wind power in the mix makes it necessary to keep a large amount of backup controllable power, presumably of nuclear origin.

4 Mix'Optimization capabilities available in MIXOPTIM

Having secured a reliable method for the performance evaluation of a given energy mix, we can now proceed to optimize the mix itself, according to the three selected criteria "economy", "environment" and "supply security" defined above, under the constraint of load following. Given the rapidity of the Monte Carlo calculation, we have used different optimization methods, either of the "simulated annealing" type or of the "genetic algorithm" type. This optimization operation provides a Pareto front (figs. 14 and 15), showing clearly that the three performance criteria are mutually incompatible.

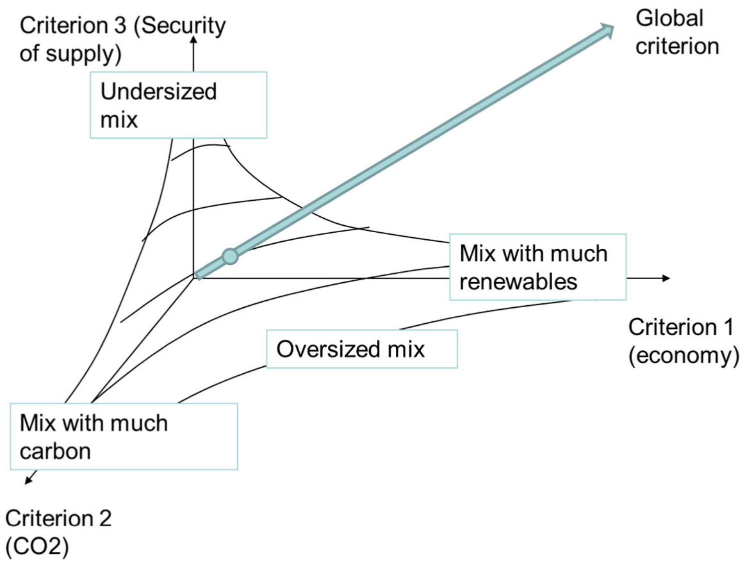

Fig. 14. The Pareto front of multi-criteria optimization. The blue arrow corresponds to single criterion optimization. The optimum mix according to this single criterion is found at the intersection between the arrow and the Pareto front.

The three criteria highlight three families of mixes:

- Mixes that are rich in renewable energies perform very well from the CO_2 viewpoint, but they perform poorly from the economic viewpoint. They are bad also from the supply security viewpoint, unless they are oversized in capacity or backed up by other sources, which brings us back to climate and economic problems.

- Mixes that are rich in hydrocarbons perform reasonably well from the economic and supply security viewpoints, but not from the environmental viewpoint, as they produce large amounts of CO_2.

- Mixes that are rich in nuclear energy perform rather well in terms of economy, CO_2 and supply security.

The MIXOPTIM software enables one to go beyond these rather obvious considerations, by providing an *objective* and *quantitative* method of optimization.

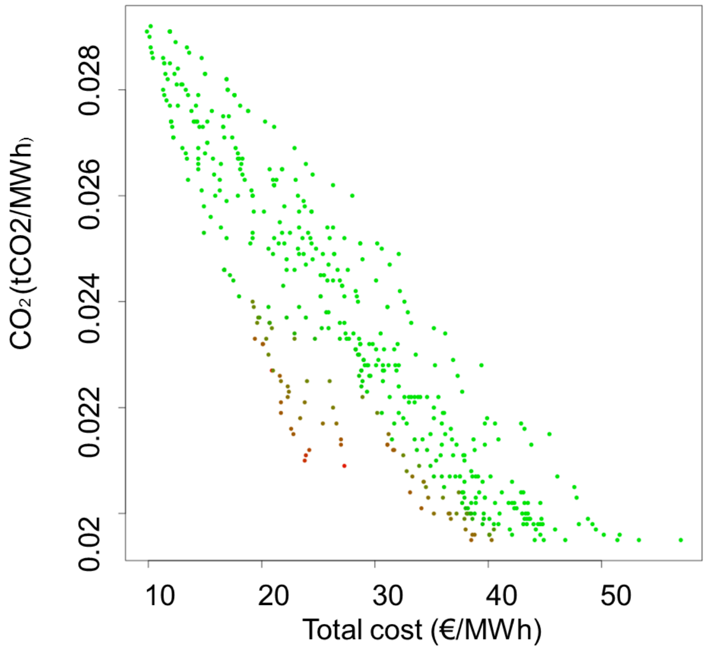

Fig. 15. The position of a sample of mixes in the three-dimensional criteria space. The cloud of points displays a Pareto front, shown as a surface in the criteria space. Here: projection of the Pareto front along the two criteria "cost" (in abscissa) and "CO_2" (in ordinates). Red dots correspond to mixes with a low degree of energy independence. The total cost and CO_2 are low, but power imports are high. The first two criteria are good (low value) but the third one is bad (high value). The domain outside the Pareto front corresponds to sub-optimal mixes; the domain inside the Pareto front is inaccessible; the shape of the Pareto front shows that the three chosen criteria are mutually incompatible: minimizing one criterion obliges one to accept large (bad) values for the other two.

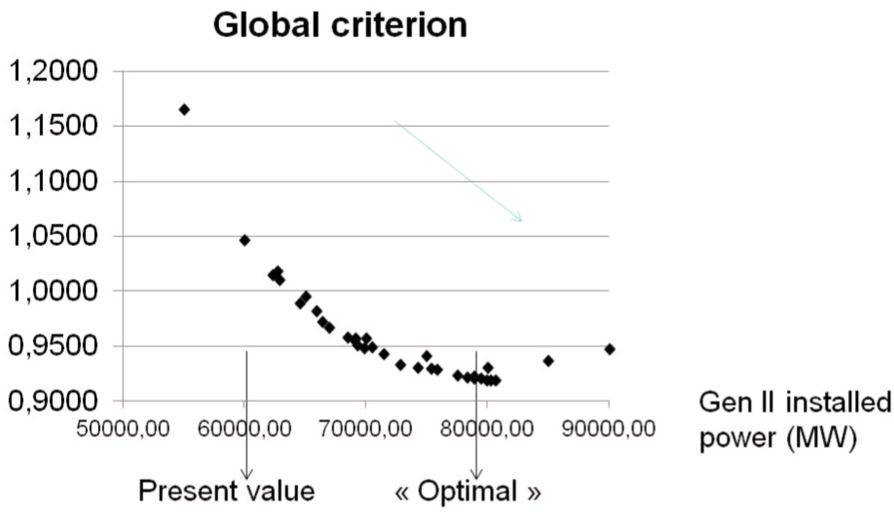

Fig. 16. Evolution of the global performance criterion of the mix as a function of the installed Gen II nuclear power. The value of 88000 MWe of installed power corresponds to the convergence value of the MIXOPTIM software.

A single criterion of optimization has been selected, as a weighted average of the three above criteria. The initial criteria were normalized to one on a given reference mix corresponding to the 2011 French case, and the global criterion was constructed using a (0.4, 0.3, 0.3) weighing of the economic, climate and security of supply criteria, respectively:

$$P_{global} = 0.4\frac{P_{economy}}{P_{eco\ ref}} + 0.3\frac{P_{CO_2}}{P_{CO_2\ ref}} + 0.3\frac{(P_{supply1} + P_{supply2})}{P_{supply\ ref}}\,.$$

The result of the optimization is shown in fig. 16.

This "optimal" mix has a total installed capacity of 139000 MW, *i.e.* 2.4 times the average demand on the French territory. The first message coming out of this optimization is thus: do not undersize the installed capacity!

This "optimal" mix contains a large amount of nuclear power, and a small amount of fluctuating energies, wind and solar. The second message is thus: nuclear power remains an important element to achieve a multi-criteria mix optimization.

This "optimal" mix is strongly export-oriented. The third message is thus: given the present prices for exported power, it is economically advisable to oversize slightly the mix, just for export purposes.

Other conclusions can be drawn from this study. Mixes with a large component of renewable (wind, solar) must be stabilized by a large proportion of backup sources, because of their intrinsically fluctuating nature. This obliges the territory to greatly oversize the total installed power, with adverse consequences on the economical performance of the mix. Moreover, the load factor of these backup sources decreases when the proportion of renewable sources increases, a factor which might act as a strong deterrent for the installation of these sources if they are run by private companies.

5 Conclusions

The MIXOPTIM software provides an objective and quantitative method to evaluate the performance of a given electricity mix, and eventually to optimize it thanks to multi-criteria optimization algorithms. The main added value of MIXOPTIM is the description of the intermittence (with an *ab initio* determination of the load factor of the sources) and quantitative criteria for indicating the ability of the mix to follow-up the power fluctuations imposed by the demand and by the mandatory sources.

We are aware that the method proposed here to evaluate the performance of an electricity mix is a crude one. Power utilities may have similar or more sophisticated tools, but they are not readily available, and not designed to optimize the mix in the general interest of the public. Moreover, the recent split between power production, transport and distribution utilities dilutes the responsibility for a global optimization of the mix. We still think that, crude as it is, the MIXOPTIM software can be a useful tool, not only for the specialists, but also for the public.

MIXOPTIM can be seen as a contribution to the public debate on energy. The software is very easy to use. It is available for free on the internet at `http://app.mixoptim.org`. The input data used in this study can be criticized. However, we are convinced that the main trends and conclusions exposed here are stable with respect to reasonable changes in the input data. The interested user can change them easily, perform his own simulation and draw his own conclusions.

MIXOPTIM is an open source code. It is technically feasible and planned to include new features, for example the contribution of externalities such as those stemming from the health impacts of producing, distributing, and consuming energy. As it is already implemented, it would be interesting to study the influence of a carbon tax on the cost of a MWh. Any contribution in this direction is welcome.

The authors are indebted to their colleagues from the Alliance Nationale pour la Coordination de la Recherche sur les Energies (ANCRE), as the debate in this circle was the seminal point for the development of MIXOPTIM. Useful contributions and encouragements from Jean-Paul Hulot, Daniel Heuer, Christian Lebrun, Jean-Guy Devezeaux, Patrick Criqui, Hubert Flocard, Jean-Baptiste Thomas, Alex Müller, Eric Proust, Frank Carré and Lynne Strudel are also gratefully acknowledged.

References

1. RTE website, `http://www.rte-france.com/fr/developpement-durable/eco2mix`.
2. *Projected costs of generating electricity*, report OECD/NEA 2010.
3. H. Flocard, J.P. Pervès, Comm. Part (data extracted from the RTE website).
4. Henri Safa, Rev. Gén. Nucl. **2**, 100 (2012).
5. The chronicles of the electricity price are available on the POWERNEXT website, `http://www.powernext.com/`.
6. All data on the French electricity production and consumption are openly available at the French RTE website, `www.rte-france.com`.
7. F. Wagner, Eur. Phys. J. Plus **129**, 20 (2014).